IALA Guideline No. 1065

on

Aids to Navigation Signal Light Beam Vertical Divergence

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Document Revisions

Revisions to the IALA Document are to be noted in the table prior to the issue of a revised document.

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<td>October 2013</td>
<td>Section 5 edited, section 7 and annex 3 added</td>
<td>Guidance added on vertical divergence for fixed light sources.</td>
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Aids to Navigation Signal Light Beam Vertical Divergence

1 PURPOSE

The purpose of this document is to assist the user with the specification of the vertical divergence of a lantern selected for a particular Aid to Navigation. This may be a fixed platform or a floating platform such as a buoy or lightvessel.

The Technical Note following the IALA Recommendations on the Determination of the Luminous Intensity of a Marine Aids to Navigation Light (E200) states that “the usefulness of a marine aid-to-navigation light depends greatly on the proportion of time for which it may be seen at any required distance”.

Moreover, the same text points out that “in enclosed waters the maximum distance at which it (the light) is required to be seen may be limited, but it is of interest to guarantee this range in as poor a visibility as it is economical to provide for”.

The objective for the user is guidance on the lantern divergence for any given aid to navigation. This may be a floating aid application, considering the motion response of the platform. It may be a fixed platform, considering focal plane height, the maximum and minimum range of visibility required, and the bridge height of ships using the aid.

2 BACKGROUND HISTORY – BUOY LANTERNS

Early in the twentieth century, electric buoy lanterns were unknown, and the illuminant was usually acetylene or propane-butane mix. These fuels were burned in an incandescent mantle, or burned in air. In both cases the light source was of the order of a centimetre in size, or larger. Lanterns at that time used pressed glass lenses, or sometimes cut glass lenses. The effect of the large light source was to provide a very wide vertical divergence, which when combined with the stable performance of the steel buoys then used, ensured that the light was visible under most service conditions.

When electric buoy lanterns started to become widely used at about the middle of the last century, most were powered by large primary battery packs. In some cases these battery packs used air depolarised cells. The battery packs were typically made of a size and shape suitable to fit into a buoy pocket which was originally designed to hold four AK-50 acetylene cylinders. These battery packs were heavy and expensive and the designers of the new electric lanterns wished to obtain a sufficient range of light for minimum power drain.

As a result there ensued a narrowing of the vertical divergence curve of the lanterns used, with the lanterns achieving good nominal range figures, but having less divergence than the gas powered lanterns they replaced. Later, plastic buoys appeared on the market, GRP at first, and these proved more lively than the older steel designs. Towards the end of the century, with an eye to reducing capital costs of buoys, some authorities specified smaller buoys than might have been used in the past.

When the IALA buoyage system was introduced globally, in 1980, there was a recognition that the complex south and west cardinal characters would demand more vertical divergence than was provided by the electric lanterns then in use, and some authorities therefore specified wider divergence. (This was prompted by an understanding that recognition of the longer and more complex characters for the south and west cardinal marks would be more affected by buoy motion.) Despite this, the divergences used were still narrow compared with the angles through which most buoys, even the more stable ones, moved during normally encountered wind and wave conditions.
Although the matching of the optical characteristics of a buoy lantern to the dynamic characteristics of the buoy on which it is mounted has been a consideration of AtoN engineers for one hundred years, it is only in recent years that IALA has started to consider how it might issue guidance on this subject.

3 SCOPE

This guideline is limited to a discussion of the light source vertical divergence, range and focal height. It does not advise on the selection of a floating platform size or shape to suit a particular geographical location.

A light intended for use on an AtoN is generally a lantern for which there is sufficient photometric data available. The luminous intensity and the spatial distribution of the emitted light are the useful parameters for establishing its nominal range. The main factors affecting its visible range are external to the installation and may be grouped as follows:

- Atmospheric light transmission (Meteorological visibility);
- Human perception (Threshold);
- Background lighting or rival lights;
- Geographical range (height of observer and light);
- Dynamic effects (for floating aids):
  - Motion of the floating platform, and the lantern light spatial distribution;
  - Variation in geographical range due to waves and obscuration by wave crests.

The first three factors are covered in other IALA documentation, specifically the E-200 series of Recommendations, and so are not discussed in detail here. This document will concern itself with the following.

- Factors affecting recognition of the signal by the observer;
- Geographical range;
- Dynamic effects on floating aids.

4 FACTORS AFFECTING RECOGNITION OF A SIGNAL LIGHT

4.1 Light signal recognition

For identification of an AtoN with a flashing light, the mariner can observe two properties of the signal from the lantern during night-time:

- colour of the light;
- character of the light.

In most circumstances, he will observe the colour first, white, red, green, yellow, blue, or maybe a combination. This will provide the first information useful for navigation. For example, if the mariner observes a red or green light, from a floating aid, then he will know that it is a lateral mark. For further information he must recognise the character of the light. For fixed and floating marks with white lights it is essential for the mariner also to recognize the character of the light in order to have any navigation information.

4.2 Character recognition sequence

The process of recognizing the character of the light can be divided into three steps:

1 Detection: the mariner observes the signal;
2 Identification: the mariner identifies the character of the signal;
3 Confirmation: the mariner identifies the character of the signal once again, to verify;
NOTE  This is the temporal sequence and assumes that the signal is not obscured from view at any time during this sequence.  The effects of human perception, atmospheric transmissivity, and other factors are assumed to not affect the visibility of the signal at the mariner’s eye.

4.3  Time required for recognition of the character of the light

There are two important time factors which define the time of recognition of the character. These are:

1  Character duration (Tc)
2  Observation duration (To)

![Character Recognition](image)

The mariner therefore needs at least two character duration periods (2 x Tc) to recognize the character of the light.

5  GEOGRAPHICAL RANGE

The geographical range is  “the greatest distance at which an object or a light source could be seen under conditions of perfect visibility, as limited only by the curvature of the earth, by refraction of the atmosphere, and by the elevation of the observer and the object or light”.

As the observer moves further away from the source, there will come a point where the light is obscured by the Earth. This is illustrated in Figure 2.

![Effect of exceeding geographical range](image)

This maximum distance is determined by the equation:

\[
R_g = 2.03 \times \left( \sqrt{h_0} + \sqrt{H_m} \right)
\]

(equation 1)  Geographical Range

where:

\( R_g \) - geographical range (nautical miles)
\( h_0 \) - elevation of observer’s eye (meters)
The factor 2.03 accounts for refraction in the atmosphere, which causes the light path from the source to the observer to be slightly curved, and also for the conversion of units between the heights in meters and the range in nautical miles. Climatic variations around the world may lead to different factors being recommended. Typical range of factors is 2.03 to 2.12.

5.1 Geographical range for floating aids

The elevation of the lantern on a floating aid directly affects the geographical range of the light. However a greater lantern height will usually require a larger floating aid, with consequent increase in capital and maintenance cost. The practical height of a light on a floating platform thus depends on the design criteria adopted.

Often these criteria consider service conditions and allow floating platforms to be grouped into general categories. An example of a classification from one authority is as follows.

- **Open sea buoys** – focal heights varying from 4 to 6 meters.
- **Moderate sea buoys** – focal heights varying from 2.5 to 4 meters
- **Coastal and Channel buoys** – focal heights varying from 1.5 to 2.5 meters
- **Restricted water buoys** – focal heights at or below 1.5 meters.

This classification is a single example, and different authorities and manufacturers will have alternative classifications, dependent on chosen design parameters and environmental conditions.

The geographical range formula (equation 1) was used to calculate the geographical ranges in the table below. It shows that for all categories of floating aids, and for an observer eye height greater than about 5 metres, the geographical range is likely to be greater than 5 miles, even for the smallest buoys. Therefore the computed geographical range will generally not be the primary limiting factor of the range of a floating aid.

The following table is reproduced from the IALA NAV GUIDE, but with the 6 metre elevation of light added. It should be noted that some major floating aids, such as lightvessels may have an elevation of light as high as 12m or more.

<table>
<thead>
<tr>
<th>Observer eye height (meters)</th>
<th>Elevation of light (meters)</th>
</tr>
</thead>
<tbody>
<tr>
<td></td>
<td>0</td>
</tr>
<tr>
<td>1</td>
<td>2.0</td>
</tr>
<tr>
<td>2</td>
<td>2.9</td>
</tr>
<tr>
<td>5</td>
<td>4.5</td>
</tr>
<tr>
<td>10</td>
<td>6.4</td>
</tr>
<tr>
<td>20</td>
<td>9.1</td>
</tr>
<tr>
<td>30</td>
<td>11.1</td>
</tr>
</tbody>
</table>

A special concern is the ability of the mariner to see the light at close range, when viewing angles are well above the focal plane. This is important for floating aids marking the edges of narrow channels when the vessel bridge height is great. The problem can be solved to some extent by choosing a light source with adequate vertical divergence, or by the use of a lantern which is designed for such conditions. This consideration applies equally to lights on fixed platforms and lights on floating platforms.

5.2 Geographical range for lights on fixed platforms
For a light source on a platform, the parameter $H_m$ is measured from the water surface to the focal plane of the lantern. The elevation of the light is calculated with equation 1 for each light separately depending on the lowest height of the observer and the required geographical range.

6 DYNAMIC EFFECTS ON A FLOATING ATO N

6.1 Describing the motion of a floating AtoN

The movement of a ship is usually described according to the terms in Figure 3, below. Not shown in this diagram is a permanent heel to one side, nor the effects of a mooring system.

![Figure 3 Description of movement of a vessel](image)

**Figure 3**

NOTE Figure 3 is reproduced by kind permission of the Royal Institute of Navigation.

When the floating aid is a light-vessel, these terms are appropriate, but for most buoys the terms reduce to a smaller set, namely: -

- Heel
- Roll
- Heave
- Surge

Heel, Roll, and Heave have a direct affect on the performance of the signal light. Surge does not. Roll and Heave are induced primarily by wave forces on the aid. Heel is usually induced by the effects of quasi-steady wind and current effects, combined with mooring load and usually changes slowly compared with the wave period.

Heel, Roll, and Heave affect the visible range and recognition of the light, by lowering the focal height and/or by tilting the focal plane of the lantern away from the observer’s eye. Lowering the focal height reduces the geographical range. Tilting the plane of the light away from the observer reduces beam intensity and luminous range.

The nature of the movement of a floating aid presents a challenge for AtoN system designers. The solution may come from two complementary approaches:

- **Floating AtoN design** – choice of the most appropriate floating platform and mooring, primarily to minimise the angles of heel and roll;
- **Lantern design** – to be appropriate to the angles of roll expected for the chosen design of floating AtoN and mooring.
6.2 Effects on Geographical Range of Waves and Floating AtoN Heave

Heave, and the position of the floating AtoN on the wave profile (e.g. crest or trough), will have the effect of altering the geographic range. In most practical cases this effect is small, and not generally considered by AtoN system designers.

6.3 Effect on Light Recognition of Waves and Floating AtoN Heave

Heave, and the position of the floating AtoN on the wave profile (e.g. crest or trough), may lead to the light being obscured from the observer when the floating AtoN is in a wave trough. This can lead to the light signal not being visible, or to the observed character being disrupted.

6.4 Effects on Light Recognition of Angle of Roll of the Floating AtoN

To picture the effect of buoy motion, imagine a buoy carrying a marine lantern, and moving under the influence of tide, wind, and wave, and constrained by the force exerted by its mooring. The Figures below show some typical effects and the attitude adopted by the buoy as a result. For simplicity, the mooring is not shown. The lantern only emits light in a disc; that is it has vertical divergence of zero.

![Figure 4](description_of_symbols)_vertically

At Figure 4, the buoy is at rest, and being perfectly manufactured, and with the mooring force acting vertically through the centre of gravity and the centre of buoyancy of the buoy, the buoy stays perfectly erect. The light disc, shown by the broken line, is horizontal.

![Figure 5](buoy_at_rest_tilted)

At Figure 5, a tidal effect has caused the buoy to move from its initial location, and also to lean at an angle of $\alpha$ under the forces of the water and the mooring. It is now apparent that the light disc is inclined by the same angle $\alpha$.

![Figure 6](buoy_angular_movement)
At Figure 6, wind and wave are adding their effects, and the buoy inclination is now modulated by a dynamic movement of $\pm \beta$. This means that the buoy, and therefore the light disc also, moves through an angle of $\alpha \pm \beta$. Note that this effect varies with the position of the observer with respect to the planes in which the angles $\alpha$ and $\beta$ are located.

7 VERTICAL DIVERGENCE OF FIXED LIGHT SOURCES

7.1 Introduction

The vertical beam divergence is specified as the angle of the beam between points where the intensity has fallen to a fraction of the maximum intensity within the beam.

The vertical divergence is typically specified between the first points where the intensity falls to 50% of the maximum. See reference [1] for more detail. Figure 7 shows the plot of intensity against vertical angle. The vertical divergence of this beam would be given as max $+1.03^\circ$, min $-1.35^\circ$ and total $2.38^\circ$.

When the lantern is used on a buoy or lightvessel, its angle is constantly changing due to the wave action and thus the beam is not always pointing towards the horizon, a wide vertical divergence is clearly an advantage.

7.2 Divergence Requirement

If the lantern is used on a lighthouse where it’s position is fixed, the angle from which the observer views the light source is dependent upon the source height, the observer height and the distance between them. By selecting a source and observer height the viewing angle for given distances can be calculated and plotted and Reference [2] Table 1 presents the minimum intensity required for a given distance.

From the analysis of vertical divergence of fixed lights found in ANNEX C it can be seen that the typical specification of a lantern’s vertical divergence does not necessarily confirm it is seen at all intended distances. The vertical beam plot of a measured lantern should enclose all of the “angle to the observer” plots to confirm suitability.

This analysis also indicates that the minimum requirements for a lantern with a vertical divergence of $\pm 0.25^\circ$ at 50% intensity would meet most requirements, although a margin should be applied to allow for errors due to assumptions and a factor of safety.
8 DEVELOPMENT OF USEFUL VERTICAL INTENSITY PROFILES

Studies cited in the appendices indicate that the buoy lantern divergences are often too small for the heel and roll angles experienced.¹

8.1 Basic Profiles

Now imagine a more conventional lantern with non-zero divergence. If an observer at some distance from the buoy is to see the light, the buoy lantern should emit light with a vertical divergence of ±(α + β).

If the distance of the observer is near the limit of visible range of the buoy lantern light, then it is clear that the vertical divergence curve of the light should be a square function as shown in Figure 8.

![Figure 8](attachment:image.png)

Remark: the angle 0° corresponds to the horizontal reference plane according to IALA Recommendation E-200-3 on Marine Signal Lights - Measurement.

The realisation of such a profile, with sharp edges as shown, might be difficult in practice, but should be achievable using modern optical design techniques. However it may not be desirable to have such a sharp cut off at the angular limits.

8.2 Complex profiles

Now take the analysis a step further. In the past, the maximum range of a light was a primary consideration of the Aids to Navigation engineer. Nowadays, buoy (and beacon) lanterns are generally used for navigation at moderate distances, for example in confined channels, and at close distances, when a ship is passing the buoy but the mariner wishes to check how close the ship passes. This adds two further considerations for deciding on a vertical divergence.

The three design requirements for the aids to navigation engineer might now be, for example:

- Requirement 1 - At long range, the buoy light should be visible despite the motion of the buoy, even in heavy sea conditions, but perhaps in very heavy seas the buoy light may not be visible.
- Requirement 2 - At medium range, the buoy light should be visible, even in very heavy seas. This means that the angle β will be greater than in 1 above.
- Requirement 3 - At very short range, the light should be visible from a close passing ship with considerable bridge height. In this case the bridge height and the proximity

of the ship to the buoy, mean that the observer is viewing the light at a great angle, see Figure 7.

Example of the navigation origin of these three requirements could be as follows.

1. Picking up the start of a buoyed channel at long range
2. Navigating in a buoyed channel, where the buoys are well separated
3. Passing a buoy or pair of buoys

Each separate requirement generates a Basic Profile, and when these three Basic Profiles are combined, the result is a Complex Profile.
8.3 Further considerations

In practice, the AtoN designer may prefer a smoother profile than shown in Figure 8.

All profiles above are minimum values. It is also desirable to define a maximum value for the luminous intensity, which should not be exceeded. A maximum value is desirable in order to harmonise AtoN performance and minimise light pollution. Thus for some applications it makes sense to define a maximum profile, so the measured intensity distribution should be inside two boundary curves.
9 DESIGN PROCEDURE FOR LANTERN VERTICAL DIVERGENCE

In the past, lantern vertical divergence was defined as the angular separation of the upper and lower 10% intensity points in the intensity profile (FWTM). More recently, lantern divergence has been defined by the Full Width Half Maximum (FWHM) intensity, the angular separation of the upper and lower 50% intensity points in the intensity profile.

This thinking on vertical divergence was based on traditional light sources and lenses that emitted a significant proportion of the light at large divergence angles. Modern technology, typified by LED lanterns, has made it possible to concentrate the light beam, such that there can be a more rectangular profile. It should also be noted that FWHM is a relative value based on the maximum intensity of the lantern; it is not related to candelas or nominal range. Therefore new performance requirements for the vertical divergence of lanterns are required as detailed below.

9.1 Floating Platforms

In considering the operational requirements of the light for a floating platform, the following should be used to define the required vertical divergence profile:

- the expected environmental conditions;
- the resultant motion of the light platform;
- the expected bridge heights of the user vessels.

9.1.1 A typical set of navigational needs and the resulting required buoy lantern performance (example)

It is useful to take the theoretical study above, and develop it further, to define the operational need and the resulting required lantern performance. The following table is for night time navigation without background illumination, and the ranges and transmissivity figures are selected for a typical example.

<table>
<thead>
<tr>
<th>Operational need</th>
<th>Environmental conditions – sea state</th>
<th>Environmental conditions - visibility</th>
<th>Observer range from buoy</th>
<th>Comment</th>
</tr>
</thead>
<tbody>
<tr>
<td>Requirement 1, Picking up the start of a buoyed channel at long range</td>
<td>Heavy seas</td>
<td>Visibility likely to be encountered 90% of the time.</td>
<td>4 M</td>
<td>The ship is approaching the start of the channel. It is acceptable for the visible range to be reduced below 4 M in poor visibility and or very heavy seas.</td>
</tr>
<tr>
<td>Requirement 2, Navigating in a buoyed channel</td>
<td>Very heavy seas</td>
<td>Visibility likely to be encountered 98% of the time. Includes heavy rain</td>
<td>1 M</td>
<td>Mariner is now in the confined channel, and must see the light, as he approaches the buoy or pair of buoys.</td>
</tr>
<tr>
<td>Requirement 3, Passing a buoy or pair of buoys</td>
<td>Very heavy seas</td>
<td>Visibility likely to be encountered 99% of the time. Includes thick fog</td>
<td>0.05 M</td>
<td>Mariner is now in the confined channel and must see the light to know his lateral position in the channel.</td>
</tr>
</tbody>
</table>

Using this set of operational needs, the parameters which will influence the lantern performance can be determined.
Table 3  Performance parameters

<table>
<thead>
<tr>
<th>Operational need</th>
<th>Sea state → buoy motion</th>
<th>Visibility → Transmissivity factor</th>
<th>Observer range from buoy</th>
<th>Comment</th>
</tr>
</thead>
<tbody>
<tr>
<td>Requirement 1, Picking up the start of a buoyed channel at long range</td>
<td>Heavy seas, $\alpha + \beta = 10^\circ$</td>
<td>Visibility likely to be encountered 90% of the time. $T_M = 0.74$</td>
<td>4 M</td>
<td></td>
</tr>
<tr>
<td>Requirement 2, Navigating in a buoyed channel</td>
<td>Very heavy seas, $\alpha + \beta = 20^\circ$</td>
<td>Visibility likely to be encountered 98% of the time. Includes heavy rain $T_M = 0.223$</td>
<td>1 M</td>
<td></td>
</tr>
<tr>
<td>Requirement 3, Passing a buoy or pair of buoys</td>
<td>Very heavy seas, $\alpha + \beta = 20^\circ$</td>
<td>Visibility likely to be encountered 99% of the time. Includes thick fog $T_M = 0.001$</td>
<td>0.05 M</td>
<td>Observer is close to the buoy, and well above. Bridge heights up to 40 metre</td>
</tr>
</tbody>
</table>

The $T_M$ factor used in Requirement 1 is derived from data published for Japan. Refer to publication “Basic Theory of Aids to Navigation”, 1988, issued by Tokokai organisation (Associate Member of IALA). Table 2.1-2 on page 79 “Ranges of Visibility in Japan” shows data for visibility derived from observations throughout the coastline of Japan, and indicates an average $T_m$ of 10.2 nautical miles. A figure of $T_M = 0.74$ (10 nautical miles visibility) is thus used in Requirement 1. The 0.223 figure is for meteorological visibility of 2 km.

Now the performance required of the lantern can be computed.

Table 4  Lantern performance

<table>
<thead>
<tr>
<th>Operational need</th>
<th>Lantern vertical divergence range</th>
<th>Input data to intensity calculation</th>
<th>Computed intensity $I_{computed}$</th>
<th>Photometric Intensity $I_{photo}$ (*)</th>
</tr>
</thead>
<tbody>
<tr>
<td>Requirement 1, Picking up the start of a buoyed channel at long range</td>
<td>0 to $\pm 10^\circ$</td>
<td>$T_M = 0.74$ $D = 4$ M</td>
<td>36.6 cd</td>
<td>$\approx 70$ cd</td>
</tr>
<tr>
<td>Requirement 2, Navigating in a buoyed channel</td>
<td>$+10^\circ$ to $+20^\circ$ &amp; $-10^\circ$ to $-20^\circ$</td>
<td>$T_M = 0.223$ $D = 1$ M</td>
<td>3.1 cd</td>
<td>$\approx 5.8$ cd</td>
</tr>
<tr>
<td>Requirement 3, Passing a buoy or pair of buoys</td>
<td>$+20^\circ$ to $+90^\circ$</td>
<td>$T_M = 0.001$ $D = 0.05$ M</td>
<td>0.0024 cd</td>
<td>$\approx 0.0046$ cd</td>
</tr>
</tbody>
</table>

(*) Remark: The computed intensity is the effective intensity under service condition. For photometric measurements the calculation of the effective intensity and the service condition...
factor has to be considered. If the shortest flash has a duration of at least 0.5s and a service
condition factor of 0.75 is used the photometric intensity $I_{\text{photo}}$ can be calculated (see IALA
Recommendation E-200-4 on Marine Signal Lights - Determination and Calculation of Effective
Intensity):

$$I_e = \frac{I_0 \times t}{a + t} = \frac{0.5}{0.2 + 0.5} I_0 = 0.71 I_0$$

(equation 2) Effective intensity (Blondel-Rey)

$$I_{\text{photo}} = I_0 / (0.71 \times 0.75) \approx 1.9 I_0$$

(equation 3) Photometric Intensity, including Blondel-Rey and service conditions factor

Where:
- $I_e$ is the effective intensity
- $I_{\text{photo}}$ is the measured instantaneous intensity

This formula only applies to rectangular flash shapes.

9.1.2 Significance of the example

The example given above is intended only to demonstrate the theory of this method of deriving
lantern vertical divergence requirements. It should not be viewed as an IALA specification for
buoy lanterns, but should be considered as indicating a design process. The process shown
relates the operational performance requirements to the lantern intensity and vertical
divergence needed to meet those requirements, via consideration of visibility conditions and
buoy motion.

It is up to the designer of the AtoN to ensure that he is aware of the operational needs, and also
to have available the meteorological and hydrological data, the visibility data, and the buoy
motion data he needs to carry through the analysis and reach the correct conclusion.

The vertical divergence profile of a lantern meeting the operational needs is therefore as
follows.

![Figure 13 Developed intensity curves (minimum values)](image)
9.2 Fixed Platforms

For fixed structures, the effects of tide, wind, and wave are assumed to be negligible, and so only geometrical effects arising from the relative heights of the light and the observer, and their distance apart, need be considered. These can be extreme, for example in the case of a light located on a tall cliff, but also used by small vessels passing close. Another common case is that of a larger ship, perhaps a bulk ore carrier, passing close to lighted pile beacons in a narrow channel.

In the latter example it is important that the light can be seen from the bridge or bridge wing of the ship as the ship passes the beacon, or passes through a pair of beacons forming a gate. In this example light from the lantern must be visible, perhaps under conditions of poor transmissivity, at the acute viewing angle from the bridge. However the light intensity required may not be great, as the distance is short.

For many lights (e.g. direction lights or sector lights) the minimum distance of approach is much greater than the height of the light or the observer. The minimum vertical divergence in this case mainly depends on the different heights of the observers.

![Diagram of light on a fixed platform and two vessels](image)

**Figure 14 Influence of observer height**

The required vertical divergence angle $\alpha_r$ can be calculated by (Figure 14)

$$\alpha_r = \arctan \left( \frac{H_{\text{max}} - H_{\text{min}}}{D_{\text{min}}} \right)$$

(equation 4) Calculation of required vertical divergence angle

Because the vertical angle is calculated for the distance at nearest approach $D_{\text{min}}$, it is not necessary to have the full intensity over the required divergence angle.

The required divergence angle could represent the required value for Full Width Half Maximum (FWHM) or even Full Width Tenth Maximum (FWTM).

When the required luminous range or the vertical position of the light is very high, it makes sense to tilt the reference axis (or reference plane if possible) of the light to compensate for the effect of the curvature of the earth (Figure 15). For an omnidirectional light the reference plane becomes a conical surface.
Figure 15  Compensation of earth curvature

For many smaller lanterns it is impractical to alter the relative position of light source and lens to achieve a conical reference surface. Therefore the design required divergence angle should be calculated by (Figure 16):

\[
\alpha_r = 2 \times \arctan \left( \frac{H_L - H_{\text{min}}}{D_{\text{min}}} \right)
\]

(equation 5)  Further calculation for divergence angle

As before \( \alpha \) represents either FWHM or FWTM, where it is assumed that the vertical beam profile is symmetrical.

Figure 16  Divergence for a horizontal adjustment

For lights that need to be used both at large distances and when passing close by, different intensities are required for different vertical angles (Figure 17).

Figure 17  Required vertical angles depending on distance

An analysis of the required intensities and angles lead to a vertical intensity distribution similar to Figure 13.
10 REFERENCE


ANNEX A  STUDIES


It is not within the remit of the authors to edit the following report. However, it should be noted that some of the terms used do not conform to the definitions given in section 4.2.

This paper was presented by the U.S. Coast Guard and describes a practical method of measuring floating platform movement by digitalizing video images and measuring angles. The aim was to determine the range at which a mariner had an 80% probability of detecting the signal (by observation of its light).

The 80% probability of detection (POD) was part of the USCG design criteria for floating aids. The other assumptions were a 10-mile standard atmospheric visibility and no background lighting. The researchers recorded twelve data sets gathered from different buoys, all of them located in open sea environments, except one river buoy. The samples were considered representative of the different buoy models in use by USCG at that time.

The graph to the right shows how the combined effect of flashing light and buoy roll sum up to reduce the POD of a light on a floating aid. It is an actual 30-second roll recording of a buoy equipped with a red 155 mm lantern operating with 12V/1.15 Amp lamp. Steady light (the middle graph) would be seen flashing or flickering and the observer would detect only the first two flashes of the actual flashing light (the bottom graph). The remaining three flashes become invisible by the buoy motion.

Another important result was the unexpected list varying between 1º and 7.4º. The researchers could not find a direct correlation between those lasting tilts and tidal currents or other likely effects and concluded that the list could not be attributed to any single dominant force (e.g. wind/buoy sail area, moorings, balancing, etc.). The causes are yet to be determined.

![Figure 18](image-url)  
*Combined effect of buoy motion and flashing light*

The overall effect on POD of lights is that a permanent list angle is to be added to the roll, decreasing further the POD. The graph below illustrates this effect.
The study demonstrates that increasing the vertical divergence of the lights has a significant impact on improving the POD, but at the expense of some loss in lantern peak intensity, resulting in an overall reduction in the maximum range.

The paper presents six graphs similar to the one below, depicting the relationship between range and vertical divergence. There are three curves. The solid line represents the loss in peak intensity versus divergence, for a fixed aid in line with the observer's eye. The composite dot/dash line is the effect of divergence versus 5.1º list and the dotted line represents the 2.6º list. It is clear that the three curves tend to align around 4 nautical miles nominal range for vertical divergence angles above 10 degrees. The study concluded that for the majority of samples, this alignment occurred for divergences of 7º to 10º.

In some graphs, the improvement of range in the presence of list continues well above 10 degrees, but the paper pointed out that it would not be practical to have lanterns with divergences tailored to each application.
The paper presented the following conclusions:

“The designed buoy nominal ranges are never achieved with the slightest amount of buoy movement or list. Buoy movement and list are present in all but the calmest atmospheric and sea conditions.

“The following general observations and trends are supported by the results:

* Nominal range is not a realistic measure of the detection range of a lighted buoy.
* Buoy list, more than buoy motion, is a severe problem, which merits further study.
* Increasing the lens divergence can significantly increase the 80% POD range of a buoy.”

The second study, presented by the German Federal Waterways Administration – Traffic Technologies Centre (TTC) compares vertical divergence of different lanterns for general aids to navigation use and also presents results for buoy movement measured in the North Sea.

The conclusions on buoy motion were that “the reliable luminous intensity for 90% of the time is the value at 11.3° of the vertical intensity distribution”. This is very close to the 10º divergence described in the previous USCG study. Furthermore, the researchers discovered that “there is a slight probability (7%) that the buoy tilts (heels) to an angle of 30°”. The Buoy Roll Angles X time graph of the USCG study records nearly 40º overall roll angles.

The following set of graphs, extracted from the original document, illustrates the findings:

**A – Different Light Sources**

![Graph A - Different Light Sources](image)

**B – Different LED Lanterns**

![Graph B - Different LED Lanterns](image)

In column A, above, one can see the comparison between LED, Gas and standard 155 mm electric lantern samples. Below, there is the corresponding variation in luminous range for a fixed light under nominal visibility conditions of V = 10 M. The column B shows the same results for LED lanterns with three different vertical divergence curves.

3 Buoy Motion Effects on Character Recognition (Trinity House)

Further studies have been carried out by Trinity House concentrating particularly on the effect of the dynamic motion of the floating platform on light recognition. In a simplified analysis of buoy motion, and in agreement with the other studies, the major effects are caused by heel and roll. The motion of a 3-metre diameter steel buoy was recorded by video and subsequently analysed to give the angle of roll plot shown as the lower curve in the graph below.
3.1 **Small vertical divergence (FWHM = 7°):**

The lantern used for this analysis was an LED lantern with a divergence profile and range versus angle curve as follows, with 7° total vertical divergence at the 50% points.

![Lantern Data Used in Trinity House Study - 7° vertical divergence](image1)

The visible range of the lantern is plotted as the upper curve in the graph below. This was obtained by relating the angle of heel of the buoy with the lantern intensity at that angle as measured in the indoor light range, and then a calculation applied to convert intensity to luminous range.

![Buoy Attitude and Range of Lantern in Waves, from Trinity House Study](image2)

The results of the analysis above were then used as an input into a study to compute the apparent character of a lantern, as observed from a distance of four miles, in the worst possible location with respect to the buoy motion. A south cardinal character is represented as a series of dots and dashes at the top of the graph shown below. The apparent character (at the assumed four mile observation point), as modified by the buoy behaviour, is shown as a second series of dots and dashes. In this example, it is apparent that much of the character is lost. It is also apparent that, over the observation period of one minute, the character of a south cardinal could be detected, and possibly identified, but not confirmed.
It is important to note some assumptions that have been made in order to simplify the study.

- The observer is stationary, not closing in on the buoy
- No allowance is made for the intelligence or experience of the observer.

**Effect of Waves on Recognition of Character**

![Figure 24](image1)

**Figure 24**

*Effect of Buoy Motion on Recognition of Character, at 4 Miles Range, for Lantern with 7° Vertical Divergence, from Trinity House Study*

### 3.2 Large vertical angle (FWHM = 21°):

Trinity House then repeated this analysis using data from a lantern of wider vertical divergence, 21° total. The intensity data were corrected to give the same area under the intensity curve as the first, 7° vertical divergence, lantern, and the effect on character recognition was determined, again for the assumed observer at a range of four miles.

**LR0610 White LED Beacon, Normalised**

![Figure 25](image2)

**Figure 25**

*Lantern Data Used in Trinity House Study - 21° vertical divergence*
3.3 Comparison

This wider divergence lantern shows some improvement in recognition at four miles, over the narrower divergence lantern. However the effect is very marked if the study is repeated for the two lanterns at 3 miles range. The two graphs below clearly show the benefit of increased vertical divergence, even at the expense of lower peak intensity.
The final part of this Trinity House study was a comparison of the modified characters of north and south cardinal lanterns, set in the same location. It is clear that the two flash patterns are very different, but it is also clear that neither character could be confirmed in the methodology described in section 3 of this document. It is also indicative of the problems that may be associated with identification and confirmation of east and west cardinal characters in that they are less salient.

3.4 Conclusion

The use of Full Width Half Maximum to specify lantern divergence is inadequate for modern LED technology lenses. Lantern divergence should be chosen with appropriate consideration of likely buoy movement, and the profile of divergence should be specified. The peak intensity is not a critical measure of performance.
4 Second Study by Trinity House

This final section considers the recommendations of the TTC study mentioned above and another paper presented to IALA by Trinity House of London in 1982. The purpose of both documents was to establish an ‘optimum’ vertical profile for AtoN lanterns for buoys.

Lantern manufacturers can benefit from the following discussion for the conception and manufacture of suitable lanterns for buoys. It is also strongly recommended that they consult the original works for a complete discussion of the topics below.

The 1982 document author declared, based on his expertise that “the beam for a buoy principally in calm weather with tidal flow should be as the curve 2B and that for rough water should be as 3B”. The total spread for the two beams are 35° and 55° respectively.

The curve 1B is the result of a practical test and the author recommends the corresponding lantern for use on buoys “in still water giving a tilt not more than ±10°”.

![Figure 30](image)

**Figure 30** Target Vertical Intensity Distributions

The aim of the document is “to prepare appropriate optical designs of lenses... to yield the 2B and 3B distributions of intensity...”

This earlier work draws similar conclusions on vertical divergence to those resulting from the work, described earlier in this Guideline, by the USCG, The German Federal Waterways Administration, and Trinity House.

5 Study by German Waterways Traffic Technology Centre

To complement this conclusion on the need for a wide divergence or “flat” pattern for buoy lanterns, the TTC document, noted above, contains an interesting proposal that is worth considering as a practical pattern to be adopted for lantern design.
The graph below summarizes the proposed pattern, where it can be seen that the required luminous intensity for the nominal range ($I_{\text{nominal}}$) should be available through an angle of ±10 degrees and the curve is further defined by 40% $I_{\text{nominal}}$ and 10% $I_{\text{nominal}}$ steps.

![Figure 31 Proposed vertical intensity profile](image-url)
6 Study by North Sea Directorate, The Netherlands

The stability calculation of the 12.5 m³ buoy shows that the static heel angle, caused by 7 Bf, is already 7 degrees and a wind force of 10 Bf results in an angle of 32 degrees (see graph below). In this case wave movements are not taken in account. The 12.5 m³ buoy is a steel skirt buoy with a diameter of 3.20 m, a weight of 4 tons and a metacentric height of 0.35 m.

Even though the stability of a plastic buoy is much better than a steel one the plastic buoy will follow the movements of the waves because of the lighter weight. For this reason the angle of heel will also be important. A plastic buoy with a diameter of 3.00 m has a metacentric height of about 1.00 m and a weight of about 2.5 tons.

![12,5 m³ light buoy stability graph](image)
ANNEX B  EXAMPLE SPECIFICATIONS

1  German Intensity Specification for Beacons for large buoys

1.1  General remarks

The specifications were used and published for a European invitation of tenders in 2003 for nearly 1000 beacons. It was accepted by 4 manufacturers (3 of them were IALA Members) and received test beacons from 3 manufacturers.

The total intensity is different for each colour. The relative profile is the same for all colours.

The luminous intensity is defined with the following geometry. For the measurement the horizon is defined by the bottom of the beacon. All values are photometric luminous intensities.

1.2  Geometry

![Diagram showing vertical and horizontal angles]

Figure 33  Vertical planes according to E-200

1.3  Nominal Intensity

The nominal intensity for each colour is the minimum photometric luminous intensity of fixed light in the horizon. The nominal intensities and the minimum values for the horizontal intensities are:

<table>
<thead>
<tr>
<th>Colour</th>
<th>Nominal intensity</th>
<th>Maximum intensity</th>
</tr>
</thead>
<tbody>
<tr>
<td>White</td>
<td>120 cd</td>
<td>180 cd</td>
</tr>
<tr>
<td>Green</td>
<td>40 cd</td>
<td>60 cd</td>
</tr>
<tr>
<td>Red</td>
<td>40 cd</td>
<td>60 cd</td>
</tr>
<tr>
<td>Yellow</td>
<td>30 cd</td>
<td>45 cd</td>
</tr>
</tbody>
</table>

The minimum profiles are shown in the figures below. The measured intensity profile must lie above the minimum profile for all horizontal angles $0 \leq \Phi < 360^\circ$. 
1.4 Intensity Profiles

1.4.1 White

![Figure 34](image1)

**Minimum Intensity Profile for White**

1.4.2 Green / Red

![Figure 35](image2)

**Minimum Intensity Profile for Red and Green**
1.4.3 Yellow

Figure 36 Minimum Intensity Profile for Yellow

1.5 Mathematical description

White (120 cd):

Table 6 Minimum vertical divergence profiles for different colours

<table>
<thead>
<tr>
<th>Vertical angle</th>
<th>Function</th>
</tr>
</thead>
<tbody>
<tr>
<td>-5° to +5°</td>
<td>$I_{\text{min}}(\theta) = 120 \text{ cd} - 12 \text{ cd} \cdot (</td>
</tr>
<tr>
<td>-10° to -5° and +5° to +10°</td>
<td>$I_{\text{min}}(\theta) = 108 \text{ cd} - 9.6 \text{ cd} \cdot (</td>
</tr>
<tr>
<td>-15° to -10° and +10° to +15°</td>
<td>$I_{\text{min}}(\theta) = 24 \text{ cd} - 1.2 \text{ cd} \cdot (</td>
</tr>
<tr>
<td>-30° to -15° and +15° to +30°</td>
<td>$I_{\text{min}}(\theta) = 6 \text{ cd}$</td>
</tr>
</tbody>
</table>

Green and Red (40 cd):

<table>
<thead>
<tr>
<th>Vertical angle</th>
<th>Function</th>
</tr>
</thead>
<tbody>
<tr>
<td>-5° to +5°</td>
<td>$I_{\text{min}}(\theta) = 40 \text{ cd} - 4 \text{ cd} \cdot (</td>
</tr>
<tr>
<td>-10° to -5° and +5° to +10°</td>
<td>$I_{\text{min}}(\theta) = 36 \text{ cd} - 3.2 \text{ cd} \cdot (</td>
</tr>
<tr>
<td>-15° to -10° and +10° to +15°</td>
<td>$I_{\text{min}}(\theta) = 8 \text{ cd} - 0.4 \text{ cd} \cdot (</td>
</tr>
<tr>
<td>-30° to -15° and +15° to +30°</td>
<td>$I_{\text{min}}(\theta) = 2 \text{ cd}$</td>
</tr>
</tbody>
</table>

Yellow (30 cd):

<table>
<thead>
<tr>
<th>Vertical angle</th>
<th>Function</th>
</tr>
</thead>
<tbody>
<tr>
<td>-5° to +5°</td>
<td>$I_{\text{min}}(\theta) = 30 \text{ cd} - 3 \text{ cd} \cdot (</td>
</tr>
<tr>
<td>-10° to -5° and +5° to +10°</td>
<td>$I_{\text{min}}(\theta) = 27 \text{ cd} - 2.4 \text{ cd} \cdot (</td>
</tr>
<tr>
<td>-15° to -10° and +10° to +15°</td>
<td>$I_{\text{min}}(\theta) = 6 \text{ cd} - 0.3 \text{ cd} \cdot (</td>
</tr>
<tr>
<td>-30° to -15° and +15° to +30°</td>
<td>$I_{\text{min}}(\theta) = 1.5 \text{ cd}$</td>
</tr>
</tbody>
</table>
2 German Intensity Specification for Lights for the marking of offshore windfarms

The IALA recommends to mark the ‘Significant Peripheral Structures (SPS) of a wind farm with lights with a range of not less than 5 nautical miles’ (IALA Recommendation O-139 On The Marking of Man-Made Offshore Structures).

However the calculation of the required intensity is ambiguous.

Therefore the German Administration developed a precise photometric description from the IALA Recommendation (with the tools of IALA Recommendation E-200 series on Marine Signal Lights).

It was assumed that the range of IALA-Rec. O-139 is the nominal (luminous) range of the light.

The nominal range is based on an atmospheric transmissivity $T_M = 0.7411$ and a required illuminance at the eye of the observer of $E_t = 2 \times 10^{-7} \text{lx}$ (no background illumination).

This leads to an effective luminous intensity under service conditions $I_{eff,s}$.

$$I_{eff,s} = 76.7 \text{ cd}$$

The next step is to calculate the required photometric luminous intensity $I_{photo}$ of the new light.

The service condition factor is 0.75.

The minimum flash duration is 1s, the time dependent luminous intensity is assumed to be rectangular.

$$I_{photo} = \frac{1}{0.75} \times \frac{0.2 \text{ s}}{1 \text{ s}} \times I_{eff,s} = 1.33 \times I_{eff,s}$$

(equation 6) Calculation of photometric intensity

The required photometric intensity is

$$I_{photo} = 1.33 \times 1.2 \times 76.7 \text{ cd} \approx 120 \text{ cd}$$

(equation 7) Values for equation 6

This luminous intensity was chosen as the minimum luminous intensity in the horizon.

According to the structure of the windfarm it was found that it was sufficient to supply a vertical divergence (FWHM) of about 5 degree. To avoid ambiguity this value was transformed to a minimum value of at least 50% of 120 cd at the vertical angles of +/-2.5°.

The resulting 5M-Beacon is used at all peripheral wind turbines. A 2M-Beacon is not used. To avoid light pollution there was the demand for an upper limit for the luminous intensity (maximum profile). The maximum profile was chosen in a way that it is still likely to be produced by the manufacturers.

The resulting maximum and minimum profiles are shown in the following figure.
Figure 37  
Resultant maximum and minimum profiles 

Photometric Intensity

-180 cd
-120 cd
lower limit
upper limit

-10° -5° -2,5° 0° +2,5° +5° +10°

vertical angle $\Theta$

120 cd
10 cd
10 cd
ANNEX C  ANALYSIS OF VERTICAL DIVERGENCE OF FIXED LIGHTS

1  Method

Due to refraction the light from a lantern will follow a slightly curved path. The curve will be assumed to have a fixed radius for the entirety of the path. The radius of the curved light path will be taken as 7 times the radius of the Earth. [3].

The radius of the Earth used will be 6371 km, the mean radius as taken from [4].

The validity of the above values is checked in Appendix A (see below) by using them to derive a formula presented in an IALA guideline (since the individual values are not presented).

The angle the observer views the lantern at will be calculated as: the angle of the line that is tangential to the curved path of light that intersects both the source and observer.

The angle to the observer will be calculated at various distances and plotted against the minimum intensity requirement for that distance. Appendix B and Appendix C, below, contain the derivation of the formulas used for this.

It is noted that the wave action will affect the observer’s height but this will be negligible compared to the viewing distance for all but close range distances. By allowing a margin of safety in the intensity requirement this can be neglected.

Alternatively two plots could be produced for the maximum and minimum observer heights to account for wave action. Furthermore, if already plotting two observer heights then the plots could be for the shortest observer height and the tallest observer height, thus for a given source height all observers are covered.

This concept can be extended further to produce a generalised plot covering a range of scenarios. The situations that will cause wide viewing angles are a short source height with tall observer and a tall source height with short observer. Both of these are somewhat limited by geographical range and as such only relevant for lower intensities. By using a tall source and tall observer a third series of points can be plotted giving relevant values out to larger ranges. The vertical beam plot of a measured lantern should enclose all of the “angle to the observer” plots to confirm suitability in all applications.

2  Result

Figure 38 shows the minimum intensity requirements at viewing angles for a source height of 30 m and an observer height of 10 m. It is plotted for ranges between 1 M and 18 M.

![Figure 38](Minimum Intensities at Viewing Angles. Source 30 m high, observer 10 m high)
The plotted curve for a measured source must completely enclose (up to the required maximum intensity for the particular station) the minimum requirement shown in Figure 38.

For example, Figure 39 shows theoretical measurements of a light source that does not meet the intensity requirements. The measured beam is narrow and any failures will be by very small angles. Also the beam does not cross 0°. Clearly the beam is unusual compared to practical lanterns, but it is presented only as a means to demonstrate use of the graph.

![Figure 39 Example of Divergence Failure](image)

There are two places the measured performance falls outside the minimum “angle to the observer” plotted line. These are both discussed below.

- The peak of the beam is approximately 27,000 cd. This corresponds to a range of 17 M, but at this range the observer would be viewing the lantern at approximately -0.158° (as plotted by the minimum requirement). The 27,000 cd intensity occurs at -0.148° while this intensity is detectable at 17 M, in this case it is not in the direction of the observer and as such will not be viewed.

The maximum intensity that meets the minimum requirement is 20,000 cd, corresponding to a distance of 16 M. So while the lantern could potentially be detected at a distance of 17 M the observer will only be at appropriate angles to view the light out to 16 M.

The same scenario may be described another way: When viewing at -0.158° the observer would be at a distance such that the minimum intensity required is 26,200 cd. At -0.158° the lantern actually only has an intensity of 16,000 cd, this is less than required. This concept is annotated by the dotted line.

As previously indicated the theoretical light source has a narrow beam compared to practical sources. As such the peak of the light source intensity (27,000 cd) is somewhat higher than the peak intensity that still encloses the requirement plot (20,000 cd). It is unlikely a practical lantern would exhibit such a large difference. The features discussed here have been exaggerated to enable them to be more clearly shown.

- The second area where the lantern’s measured performance falls outside the plotted requirement is between -0.133° and -0.138°. The measured intensities at these angles are lower than required for the observer to detect the light. For example, when viewing the lantern at -0.135° the observer will be at a distance requiring an intensity of 1,760 cd. The lantern’s intensity at -0.135° is 1,000 cd. This concept is annotated by the dashed line.
If the 50% divergence angle is larger than the plotted angles for all but the lowest intensities then it will likely meet requirements for practical viewing ranges. For example, a lantern with a vertical divergence of ±0.25° at 50% intensity would enclose the majority of the plotted line in Figure 38 (possibly by a large margin depending on what the 50% intensity actually is) and as the divergence at lower intensities typically increases, it would likely meet requirements at all practical ranges.

Due to the non-linear relationship between range and intensity, moving closer greatly reduces the intensity required for the observer to reliably detect the light. Thus at close ranges, where the magnitude of the viewing angles become larger, the intensity requirement is very low and changes little with angle. This is beneficial since at the larger angles, where lanterns typically emit less light, the intensity requirement is very low. Thus the shape of a lantern's beam profile naturally suits the shape of the minimum requirement.

As discussed earlier, several scenarios could be created to cover the situations requiring wider divergence and longer distances. One example of this is shown in Figure 40. Each series is plotted from 1 M up to the maximum geographical range for that scenario. To be suitable for all scenarios, a lantern's measured profile should enclose all of the plots.

3 Analysis of Divergence Profile

The shape of the plot will be analysed as a means of checking, to some degree, that the calculations correctly model the actual behaviour.

There are two distinct curved parts to the plot shown in Figure 38.

Above minimum intensities of approximately 365 cd (7 M), as the observer moves further away from the source the magnitude of the angle between the light source and observer increases. The increasing angle with range is graphically shown in Figure 41. Clearly moving away from the light source requires greater intensity to meet the range so the curve has both increasing intensity and increasing angle.
Below intensities of approximately 365 cd (7 M), as the observer moves away from the source the magnitude of the angle between the light source and observer decreases. The decreasing angle with increasing range is shown in Figure 42.

The two regions of differing behaviour are caused by the curvature of the earth. When the observer is close to the light source, moving away has little effect on the vertical position of the observer and the angle is affected primarily by the horizontal change in distance. This results in the angle reducing with increasing distance.

When the observer is further away from the light source, moving away has a larger effect on the vertical position of the observer, resulting in the angle increasing with distance.

Figure 43 illustrates the gradient of tangents at varying distances. The tangents further from the source are much steeper and thus vertical position changes rapidly with distance.
If the source height, as shown in Figure 41 is sufficiently increased, the range at which the behaviour changes from one region to the next is increased. Figure 44 illustrates this.

Between Figure 44 (a) and (b) the distance between source and observers are the same. The only difference is the source height. In (a), the further observer views the source from a larger angle. In (b), the further observer views the source from a narrower angle. Clearly the heights of source and observer will affect the shape of the plot.
Appendix A  Confirming values used with IALA Guideline

The curve radius for the refracted path of light is shown in Appendix C.

The formula and explanation for geographical range is given in (equation 1), see section 5.

The geographical distance will be calculated for a straight line and then factors for refraction and conversion of units will be added.

To simplify matters, the observer height will be considered to be 0 m. Thus

\[ R_g = 2.03 \times \sqrt{H_m}. \]

Figure 45 shows \( R_g \) and some additional parameters required to calculate it.

\[ R_g = \sqrt{(r + H_m)^2 - r^2} \]

\[ R_g = \sqrt{r^2 + H_m^2 + 2rH_m - r^2} = \sqrt{H_m^2 + 2rH_m} \]

And since \( 2rH_m \gg H_m^2 \) then:

\[ R_g \approx \sqrt{2rH_m} \]

The curved path of the light due to refraction, having a radius 7 times the radius of the Earth, may be accounted for by using an increased Earth radius, \( r' \). Where

\[ r' = \frac{7r}{6}. \] [3]

To convert from metres to nautical miles, a factor of the value in meters is divided by 1852.

A value of 6371 km is used for the actual radius of the Earth.
\[ R_g \approx \frac{1}{1852} \sqrt{\frac{2 \times 7 \times 6371 \times 10^3}{6}} H_m \]

\[ R_g \approx 2.08 \sqrt{H_m} \]

This factor of 2.08 is in the centre of the range of IALA recommended factors and shows the values used are similar to the values used to form the IALA recommendations.
Appendix B  Finding Observer Coordinates

\[ \theta_0 = \text{Angle between source and observer orientation} \]

D = Distance to observer along the Earth's curved surface.

r = Radius of Earth.

\[ Ob_x = (r + H_o) \sin \theta \]

\[ Ob_y = (r + H_o) \cos \theta - H_m - r \]

Finding \( \theta \) from the ratio of distance to circumference:

\[ \theta = \frac{D}{2\pi r} \times 360^\circ \]

Or if using radians:

\[ \theta = \frac{D}{2\pi r} \times 2\pi = \frac{D}{r} \]
Appendix C    Finding Angle to Observer

Where:
- $\theta_o$ = Angle to Observer
- A = Position of source: 0, 0.
- B = Position of observer, this has previously been found.
- C = Centre of path curve
- D = Mid-point between AB
- AC and BC both have a magnitude equal to the radius of the curve: $r'$.

The angle to the observer is:

$$\angle AC + 90°.$$  
$$\angle AC = \tan^{-1}\left(\frac{AC_y}{AC_x}\right) - 180°$$  
(-180° since the result will return first possible solution and the second is of interest. If the observer is higher than the source this is not required).

Since A is 0, 0 then

$$\angle AC = \tan^{-1}\left(\frac{C_y}{C_x}\right) - 180°$$

The coordinates of point C are now required:

$$C = AD + DC = \frac{AB}{2} + DC$$

AB is known as it is simply the position of the observer (since A is 0, 0).

$$DC_x = ||DC|| \cos \angle DC$$
$$DC_y = ||DC|| \sin \angle DC$$
\[ \angle DC = \angle AB - 90^\circ = \tan^{-1}\left(\frac{B_y}{B_x}\right) - 90^\circ \]

The magnitude of DC can be found using Pythagoras’s theorem:

\[ \|DC\| = \sqrt{r^2 - \left(\frac{\|AB\|}{2}\right)^2} \]

\[ \|AB\| = \sqrt{B_x^2 + B_y^2} \]

From these formulas all required information can be calculated. They can be combined into a single, unwieldy, equation but for simplicity were used in their intermediate form in an excel spreadsheet.